



## Credit Default Swaps with R

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### Abstract

A credit default swap (CDS) is a bilateral agreement between two parties (the protection buyer and the protection seller) with respect to default by a third party. Over the past two decades, CDS have been one of the fastest growing parts of the financial markets. First, we explain the basics of CDS along with key concepts like coupon, spread, notional, recovery rate, upfront, probability of default and the ISDA Standard Model. Second, we introduce Markit and Bloomberg, the two primary sources for CDS data and analytics. Third, we describe the credit **R** package, an open source tool which allows users to calculate information related to CDS.

*Keywords:* credit default swap, pricing, **R** package.

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## 1. Introduction

This paper explains the mechanics of credit default swaps (CDS), a type of **credit derivative** that, in exchange for payment, transfers **credit risk** from one group of investors to another. First, we introduce the concept of CDS with an analogy to housing insurance, and highlight that a CDS allows one party to purchase insurance against the default of a specific bond from another party. This insurance is usually called “protection.” We then introduce some simplified examples of one-period and two-period CDS in which a portfolio manager at Highbridge

purchases protection from Citigroup. Many important concepts are introduced: **notional amount**, **coupon** and **spread**. We then explore complications that alter CDS pricing calculations, such as **interest rates**, **recovery rate**, **probability of default**, **accrued coupon** and **upfront payment**. Further complexity arises from allowing for a non-constant probability of default and non-constant interest rates. We then consider the N-period CDS as a more accurate model of real transactions. Further complications such as calculus for the continuous case will be introduced, as a culmination for this theoretical section.

The next section focuses on real world data from Bloomberg and Markit, the two sources most often used by professional investors. For Bloomberg, we will dive into the specifics of the Deal Section, Calculator Section and Market Section to see how theoretical CDS concepts are used in practice. We then explore the Markit CDS Calculator, with an emphasis on the terminology differences from Bloomberg.

Finally, we introduce the **creditr** package, which allows users to calculate information regarding a particular CDS in **R**. The **creditr** package uses the same ISDA Standard Model that Bloomberg and the Markit CDS Calculator use, and provides generic methods to output the calculation results in a way that is similar to Bloomberg and Markit.

## 2. CDS Basics

### 2.1. An Example: Property Insurance

Consider a simpler form of purchasing protection: property insurance.

Suppose that a homeowner wants to purchase \$100,000 worth of property insurance on her house, covering the period from January 1 through December 31. For one year of coverage, an insurance company charges a fee of \$1,000. Call this \$1,000 the **premium**. In exchange for the premium, the insurance company agrees to pay \$100,000 to the homeowner if there is any property damage during that year. If damage does not occur, then the insurance company pockets the \$1,000 premium and doesn't pay anything to the homeowner.

In this simplified insurance agreement, the homeowner pays the premium on January 1, the beginning of the coverage period. If the property is damaged, the insurance company pays the \$100,000 on December 31, regardless of when the damage occurred. The interest rate is 0%.

The **expected cash flows** for the agreement depend on the probability of property damage. Since the homeowner will pay the \$1,000 premium and will potentially receive \$100,000—if the house gets damaged—the homeowner's expected cash flows are as follows. ( $P_h$  refers to

the homeowner's estimate of probability of the property damage.)

$$\text{Homeowner's expected cash flows} = - \$1,000 + (P_h \times \$100,000) \quad (1)$$

Since the insurance company will receive the \$1,000 premium and will potentially pay \$100,000— if the house gets damaged—the insurance company's expected cash flows are as follows. ( $P_c$  refers to the insurance company's estimate of the probability of property damage.)

$$\text{Insurance company's expected cash flows} = \$1,000 - (P_c \times \$100,000) \quad (2)$$

Note that  $P_h$  and  $P_c$  do *not* have to be the same. The homeowner and the insurance company may have two different estimates of the probability that the house will get damaged during the year. The homeowner doesn't know the insurance company's estimate, and the insurance company doesn't know the homeowner's estimate. In fact, an outside third party does not know either  $P_h$  or  $P_c$ .

Assume that both the homeowner and the insurance company are **risk-neutral**, meaning that they only care about expected cash flows. For example, a risk-neutral investor would be willing to pay \$1 for a 1% chance of a \$100 payment and would view the two sides — the \$1 and the 1% chance of \$100 — as equal in value. Most investors are not risk-neutral. They would prefer a guaranteed \$1 over a 1% chance of winning \$100. Assume that both parties agree to the above insurance agreement. What does this imply about  $P_h$  and  $P_c$ ?

The homeowner only agrees to the deal if  $P_h \geq 0.01$ . Any lower value would mean that her expected cash flows were negative and, by assumption, we know that neither the homeowner nor the insurance company will enter an agreement with negative expected cash flows. For the same reasons, if the insurance company agrees to the deal, then  $P_c \leq 0.01$ . Neither of these facts allow us to conclude anything about  $P_{true}$ , the true probability of damage. Either the homeowner or the insurance company or both could be wrong in their estimates. However, we can now define  $P$  — **risk-neutral estimate of the probability of property damage** — as the single value for both  $P_h$  and  $P_c$  which makes the expected cash flows identical. Mathematically:

$$-\$1,000 + P \times \$100,000 = \$1,000 - P \times \$100,000 \quad (3)$$

$$2 \times P \times \$100,000 = 2 \times \$1,000 \quad (4)$$

$$P \times \$100,000 = \$1,000 \quad (5)$$

$$P = 0.01 \quad (6)$$

The risk-neutral probability of property damage is 1%. Based on this calculation, we can determine that the homeowner's prediction for the probability of damage is at *least* 1%, and the insurance company's prediction for the probability of damage is at *most* 1%. For example, if the homeowner believes that there is a 1.5% chance that the house will be damaged, she would want to make a deal that assumes a damage probability of 1% because, in that case, the expected value of the contract would be \$500. Any belief in a damage probably greater than or equal to 1% makes her willing to agree to a deal that assumes a damage probability of 1%. Conversely, if the insurance company believes that there is a 0.5% chance that the house will get damaged, it would want to make a deal that assumes a damage probability of 1% because, in that case, the expected value of the agreement is positive.

In other words,  $P$  — the risk neutral probability of property damage — is not necessarily the *true* probability of property damage, nor is it either party's estimate of the probability of damage. The value of  $P$  is that which makes the expected cash flows equal for both sides. It is a mathematical construct with no necessary connection to the real world.

### *Property Insurance Complications*

Unfortunately, the above insurance purchase — although relatively simple — made several assumptions that excluded real aspects of the insurance market. Some of these aspects are parallel to aspects of the CDS market that we will ignore and, therefore, exclude from calculations in all sections of this paper. Below we list such assumptions:

1. We assumed that both the homeowner and the insurance company are risk-neutral when, in reality, both are likely to be risk-averse. This is especially true for individual homeowners. Taking our example, if the homeowner is risk-adverse and believes the probability of damage to be 1.5%, she would be willing to pay a coupon of more than 1.5%. Insurance companies, on the other hand, can diversify some of their risk by selling many policies.
2. We assumed that both parties would only agree to the insurance agreement if the ex-

pected cash flows were equal. As the above assumption described, an insurance buyer is generally willing to accept negative expected cash flow. Conversely, insurance companies need to have positive expected cash flows because they aim to *profit* from selling insurance; if they charge as much as they expect to pay, then they can't expect to make any money as a company, or to pay any of the overhead associated with selling insurance.

3. We discussed the insurance agreement in a way that assumed we knew if there had been damage. How does the insurance company determine that there has been, for example, a house fire when the homeowner claims that one occurred? The insurance company would probably want some form of proof, and the company might even have someone visit the house and confirm the damage.

We maintain these assumptions throughout the paper, as does most of the academic literature on CDS. Other assumptions, listed below, will be relaxed in the sections that follow.

1. The agreement considered only two cases: either the entire house was damaged (and the insurance company paid \$100,000), or the house was not damaged (and the insurance company paid \$0). However, there are other likely outcomes we need to account for. What if only half of the house was damaged?
2. Interest rates are rarely zero, nor are they constant.
3. Insurance agreements need not run exactly from January 1 to December 31. We must allow for variable length contracts.
4. Insurance payments are often made over time at distinct intervals rather than all at once at the start of the contract.

We will incorporate these extensions in the sections which follow as we move from the insurance example to actual credit default swap agreements.

## 2.2. Simple One-Period CDS

Instead of buying insurance against property damage, consider buying insurance against a company's inability to pay back its bondholders. The risk of a company being unable to meet its debt obligations is known as **credit risk**.

Let's say a portfolio manager at a hedge fund, Highbridge, believes that Alcoa, isn't likely to return the money that its bondholders have lent it. In other words, she<sup>1</sup> believes that Alcoa will **default** on its bonds. She enters a one-year CDS agreement with an investment bank — say, Citigroup — in which she purchases protection of \$100,000, the **notional amount** of the CDS. Note that bondholders can purchase protection on their bonds (i.e. an Alcoa bondholder can enter a CDS in which she purchases protection against Alcoa's default), but this does not have to be the case. In our example, Highbridge does not own Alcoa bonds.

Keep the same timeline (January 1 to December 31) and numerical figures from our property insurance example. The interest rate is still 0%. In exchange for protection on the \$100,000, Highbridge agrees to pay a **coupon** (equivalent to the premium in our property insurance example but using different terminology) of 1% of the notional amount. The coupon payment of \$1,000 is paid on January 1, the beginning of the coverage period. This side of the CDS is called the **premium leg**, and Highbridge is known as the **protection buyer**, since she is purchasing protection.

The other side of the CDS agreement is known as the **protection leg**, since it involves protection payment in case of default, and involves Citigroup, called the **protection seller**. If Alcoa defaults, Citigroup pays the notional amount of \$100,000 to Highbridge on December 31, the end of the coverage period. Alcoa is known as the **reference entity** in this CDS since the protection buyer desires protection against the risk that Alcoa will default.

Note that we are using the terms "premium" and "coupon" interchangeably. Premium is commonly used to describe the periodic fee paid by the protection buyer, which is why this side of a CDS is called the premium leg. However, even though the term coupon is commonly used to describe the periodic fee paid to a bondholder, coupon can also be used to describe the periodic fee in a CDS context. For clarity, we will just use coupon in this paper when referring to the periodic fee paid by the protection buyer.

Since the Highbridge portfolio manager makes money if Alcoa's fiscal health deteriorates, she can be said to be **short credit**. Shorting is generally a method of profiting from the deterioration of a security, such as a bond. Alternatively, since Citigroup loses money if Alcoa collapses, Citigroup is **long credit**. In other words, Citigroup's returns are similar to (or, at least, have the same sign as) the returns of a person who owns Alcoa bonds. If Alcoa does well, Citigroup does well, and if Alcoa doesn't, then Citigroup doesn't either.

The expected cash flows from the agreement depend on the probability of default. Since High-

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<sup>1</sup>For ease of reading, we designate the buyers of protection in our examples as female and the sellers of protection as male.

bridge will pay the \$1,000 premium and will receive \$100,000 if Alcoa defaults, Highbridge's expected cash flows — which represent the “premium leg” — are as follows.

$$\text{Highbridge's expected cash flows} = -\$1,000 + (P_h \times \$100,000) \quad (7)$$

Since Citigroup will receive the \$1,000 premium and will potentially pay \$100,000 if Alcoa defaults, Citigroup's expected cash flows — which represent the “protection leg” — are as follows.

$$\text{Citigroup's expected cash flows} = \$1,000 - (P_c \times \$100,000) \quad (8)$$

Note that Highbridge's expected cash flows are the same as that of the homeowner in the property insurance example, and Citigroup's expected cash flows are the same as that of the insurance company. This shouldn't be too surprising, though, since the numerical figures in both examples are the same. Also, note that the probability estimates serve exactly the same roles as before:  $P_h$  and  $P_c$  are the default probability estimates for the homeowner/Highbridge and for the insurance company/Citigroup.

We will again assume that both parties in the CDS are risk-neutral and that both parties will only enter deals in which the expected cash flows are greater than or equal to zero.

Assume that a deal is reached; both parties agree to the CDS. As in the property insurance example, we can equate the cash flows to find the **risk-neutral probability of default**, which is the value of  $P$  implied by the deal. Note that an alternate definition of this probability is the value of  $P$  at which the premium leg equals the protection leg:

$$\text{Highbridge's expected cash flows} = \text{Citigroup's expected cash flows} \quad (9)$$

$$\text{Premium leg} = \text{Protection leg} \quad (10)$$

$$-C + (P_h \times V) = C - (P_c \times V) \quad (11)$$

The left side of Equation (11) indicates that Highbridge has to pay the coupon,  $C$ , but might receive the notional amount  $V$ , in the case of default. The right side shows the inverse: Citigroup will receive the coupon payment,  $C$ , but could potentially pay the notional amount of  $V$ . The estimated probabilities of default,  $P_h$  and  $P_c$  are not necessarily the same, nor does either necessarily correspond to the true probability of default,  $P_{true}$ .

Plugging in known values ( $C = \$1,000$  and  $V = \$100,000$ ), we get:

$$- \$1,000 + (P_h \times \$100,000) = \$1,000 - (P_c \times \$100,000) \quad (12)$$

Since this is the same equation and the same numerical figures from the property insurance example, we know that, if we set  $P_h = P_c$ , then both must equal 1%. Recall that this 1% represents the “risk-neutral” value of  $P$ . Parallel to the property insurance example, we can infer (given that both parties have agreed to the CDS) that Highbridge’s estimate  $P_h$  must be at least 1%, and Citigroup’s estimate of  $P_c$  must be at most 1%. Going forward, we will ignore  $P_h$  and  $P_c$  and just focus in  $P$ , the risk-neutral probability of default.

### *One-Period Case Over Time*

Consider what happens to the simple one-period CDS over the life of its contract—specifically, how the risk-neutral probability of default (or risk-neutral  $P$ ), the **mark-to-market** value, the **spread**, and the **profits and losses** (P&L) of the Highbridge portfolio manager and Citigroup change from January 1 to December 31, assuming that Alcoa does not default during that year.

We can think of the **mark-to-market** value as the price at which the CDS would sell at any given time over the life of the contract. For example, on June 30, midway through the year, a prospective buyer would pay \$500 to Highbridge in order to replace her as the protection buyer. Since Highbridge pays the \$1,000 coupon payment on January 1 and would receive \$500 from the prospective buyer on June 30, both Highbridge and the prospective buyer pay a net amount of \$500—which makes sense since one year of coverage merits a coupon payment of \$1,000 and thus six months of coverage is worth \$500.

We can consider the **spread** to be equal to the coupon in this particular CDS (we will discuss the spread in more detail in the section regarding non-standard coupons). One difference is that we refer to coupons in percentages—in this case, 1%—and we refer to spreads in basis points (bps). 1% is equal to 100 bps.

P&L represents the change in market value of the CDS contract on, in this case, a day to day basis. A good way of thinking about the P&L of, for example, Citigroup in this CDS is to consider the fraction of the coupon payment that Citigroup *earns* each day. We know that Citigroup receives a coupon payment of \$1,000 on January 1, but Citigroup’s P&L measures what Citigroup earns each day that it provides protection during that year.

Here is a table that considers what happens to the above-mentioned variables as the contract matures from January 1 to December 31. Note that we are considering the case in which Alcoa does not default during the one-year contract.



Date	$CF_H$	$CF_C$	$P$	MTM	Spread	$P\&L_H$	$P\&L_C$
Jan 1	-\$1,000	\$1,000	1%	\$1,000	100 bps	\$0	\$0
Mar 31	\$0	\$0	.75%	\$750	75 bps	-\$250	\$250
Jun 30	\$0	\$0	.50%	\$500	50 bps	-\$500	\$500
Dec 31	\$0	\$0	0%	\$0	0 bps	-\$1,000	\$1,000

Table 1: This table measures how several variables — the cash flows (CF) for each party, the risk-neutral value of  $P$ , the mark-to-market (MTM) value of the contract, the spread, and the profit and loss (P&L) for each party — change as the simple one-period CDS matures. Since this table considers the case in which the reference entity (Alcoa) does not default, the only cash flow is the \$1,000 coupon payment from Highbridge to Citigroup on January 1. The risk-neutral value of  $P$  decreases from 1% to 0% as the contract matures. Note that this “linear” drop off in the risk-neutral  $P$  is proportional to the time remaining until the termination of the contract. For more realistic contracts, the relationship is non-linear. The mark-to-market value decreases from \$1,000 to \$0 from January 1 to December 31 and the spread decreases from 100 bps to 0 bps. Note that as Citigroup profits from providing protection coverage (gains \$1,000 by Dec 31), Highbridge loses. On January 1, the P&L of both parties is 0.

As we can see in Table 1, the risk-neutral  $P$  decreases from 1% on January 1 to 0% on December 31. On June 30, the risk-neutral  $P$  has dropped to 0.5% because only half of a year remains for Alcoa to default during the contract—and thus the risk-neutral  $P$  is half of its initial value. This demonstrates the direct relationship between the risk-neutral  $P$  and the contract duration. In our simple one-period case, we say that the coupon is 1% and the spread is 100 bps. We can observe in Table 1 that the spread decreases from 100 bps to 0 bps over the year—a reflection of the fact that, at the end of the contract, Citigroup has earned and been paid the full \$1,000 coupon payment by providing a full year of coverage. This process—the spread drop over the duration of the contract—is known as **rolling down the curve**.

### *Simple One-Period Case Complications*

Similar to the property insurance example, we made several assumptions in our simple one-period CDS that are not consistent with the CDS market.

1. We assumed that the protection buyer was risk-neutral, when she could very well be risk-adverse. She may be willing to pay a coupon larger than 1% even if she predicts the probability of default to be 1% because she strongly desires to have protection on Alcoa.
2. Also as in the property insurance example, we assumed that both parties would only agree to the deal if the expected cash flows were both equal to zero. This could very well not be the case if, for example, Citigroup is selling protection in many CDS agreements and needs or wants to make a profit.
3. In the simple one-period CDS, we assumed that the protection leg would only be paid out in the case of default and bankruptcy. However, depending on the particular CDS, the protection leg could be paid out even if the reference entity defaults and doesn't go bankrupt. In fact,

there are several scenarios that can be considered **credit events**—occurrences that merit the payout of the protection leg in a CDS. Besides default and bankruptcy, common credit events include failure to return money to bondholders within a certain amount of time, a credit rating downgrade (explained in Section 3) and the confiscation of assets, among other events.

Because we made many more assumptions and failed to address many aspects of a CDS, we have split up the following simple one-period CDS complications into five separate sections that address interest rate, recovery rate, accrued coupon, non-standard coupon and upfront payment.

### Interest Rates

Up until now, we have assumed that the interest rate is 0% in the simple one-period CDS. The **interest rate** is a benchmark rate that participants in the CDS market use to discount cash flows. How might the cash flows in the CDS agreement change if the interest rate was *not* 0%? First, look at our CDS agreement: Highbridge pays the premium on January 1 and Citigroup pays the \$100,000 (if Alcoa defaults) on December 31. Citigroup receives the \$1,000 coupon a full *year* before Highbridge would receive the \$100,000 payment if Alcoa defaults. In an environment where the interest rate is 10%, Citigroup could theoretically invest the \$1,000 sum starting January 1 and earn interest for that year:

$$\text{Coupon with Interest} = \$1,000 \times (1 + .10) = \$1,100 \quad (13)$$

Not just the coupon grows at the interest rate. A similar discounting effect applies to the potential credit loss as well. If a credit event triggers a loss in the future, then we must discount the loss amount by an interest rate  $i$  to get the present value, before we can set up the equality and solve for the risk-neutral  $P$ .

Here are the expected cash flows for a generic case. ( $P$  refers to the risk-neutral probability of default;  $V$  represents the notional value;  $i$  stands for the interest rate; and  $C$  is the coupon payment).

$$-C + \frac{P \times V}{(1+i)} = C - \frac{P \times V}{(1+i)} \quad (14)$$

How might a discounted credit loss affect  $P$ , the risk-neutral probability of default which, by definition, equates the expected cash flows? Plugging in known values:

$$-\$1,000 + \frac{P \times \$100,000}{(1+0.1)} = \$1,000 - \frac{P \times \$100,000}{(1+0.1)} \quad (15)$$

Solving for  $P$ , we get 1.1%. Since both parties agreed to this CDS in an environment where the interest rate is 10%, we can say that Highbridge's prediction for the probability of default is at least 1.1%, and Citigroup's prediction for the probability of default is at most 1.1%.

### *Recovery Rate*

Recall that in our property insurance example, we mentioned that some cases—such as a house fire that burns half of a house—may not merit a full payment equal to the notional amount (in that case, \$100,000). Similarly, when a company defaults, an **auction** occurs in which some of the company's bondholders come to sell their bonds, and prospective buyers come to buy the bonds at, usually, lower prices. We note this because the price at which bonds can be sold after default affects the protection leg payment in a CDS contract.

Suppose that Alcoa defaults during the simple-one period CDS, and a bond that had a face value of \$100 can now be sold at the auction for a price of \$55. (Note for each complication in the simple one-period CDS, we are ignoring the effects of the other complications. For example, we assume that there is an interest rate of 0% in this example.)

In this case, the **recovery rate**—the rate representing the amount of value a bond retains after default—is 55%. As such, Citigroup would only have to pay 45% of the notional amount—\$45,000—instead of the notional amount of \$100,000 because bondholders who have purchased protection are able to retain 55% of their bonds' worth.

So, we need to factor the recovery rate into our calculation of expected cash flows because it changes the value of the protection leg in the simple one-period CDS. Since Highbridge and Citigroup will only both agree to this CDS if the expected cash flows are equal, we set Highbridge's expected cash flows and Citigroup's expected cash flows equal to each other. ( $RR$  refers to the recovery rate and, as before,  $P$  is the risk-neutral probability of default,  $C$  is the coupon payment, and  $V$  is the notional value.)

$$\text{Portfolio manager's expected cash flows} = \text{Citigroup's expected cash flows} \quad (16)$$

$$-C + (P \times V \times (1 - RR)) = C - (P \times V \times (1 - RR)) \quad (17)$$

Note that the above equation is an extension of Equation (11), the only difference being that we multiplied  $V$  by  $(1 - RR)$  to account for the change in the protection leg. The protection seller only has to pay the fraction of the notional amount that the recovery rate does *not* account for (hence  $(1 - RR)$  instead of just  $RR$ ). Another term for this amount is the loss given default (LGD):

$$(1 - RR) = LGD \quad (18)$$

We'll stick to using  $RR$  instead of the  $LGD$  for now.

To solve for the risk-neutral probability of default, we plug in known values ( $C = \$1,000$ ,  $V = \$100,000$ , and  $RR = 0.55$ )

$$-\$1,000 + (P \times \$100,000 \times (1 - 0.55)) = \$1,000 - (P \times \$100,000 \times (1 - 0.55)) \quad (19)$$

After combining like terms, dividing both sides by 2 and simplifying, we get:

$$P \times \$55,000 = \$1,000 \quad (20)$$

Therefore,  $P$  is 1.8% (rounded to the nearest tenth) in this scenario where the recovery rate is 55%.

### *In the Case of a Default*

When a default does occur, the protection seller would owe the protection buyer the notional amount minus any money recovered from the company. It must be noted that the protection is effective for the credit events that have taken place since 60 days *before* the trade date; this date is known as the **backstop date**. Before the "Big Bang Protocol" in April 2009, this date used to be one day *after* the trade date. If there is a delay in awareness about a credit event, the new system allows sufficient time for the two dealers to discover and process the information. Should a bond (that is the reference obligation in a CDS agreement) default, the counterparties can compensate accordingly in two ways. The first is a **physical settlement** in which the buyer will actually deliver the defaulted bonds to seller, and the seller will then pay the face value of those bonds. The disadvantage to this particular transaction is that the buyer(s) of protection will have to find and deliver those bonds to the seller even if they don't own the bonds themselves. This may artificially drive up the price of the bonds, and is more likely to happen when there is a large number of outstanding CDS contracts.

The alternative to a physical settlement is a **cash settlement**, in which the seller simply pays the following to the buyer: notional amount  $\times$  (1 - recovery rate). Unfortunately, determining a recovery rate is often an issue. One approach the ISDA has adopted lately is an **auction** style process in which major dealers submit their bids for the value they place on a company's debt. CDS contracts for corporate bonds generally assume a 40% recovery rate for valuation purposes.

### *Accrued Coupon*

Back to our simple one-period CDS example—0% interest rate, 0% recovery rate. For this scenario, both the premium leg and the protection leg are paid on December 31. We have so far assumed that our CDS agreement between Highbridge and Citigroup begins on January 1 and ends on December 31 of the same year. However, what if this agreement wasn't made exactly on January 1? What if it was made on March 31, one quarter through the year? Using our simple one-period CDS example, this implies that Highbridge would have to pay the same coupon of  $1\% \times (\$1,000)$  on March 31 for only receiving nine months (instead of one year) of protection coverage.

Assume that both parties agreed to a deal under these conditions.

Since the risk-neutral probability of default decreases as the contract duration decreases (see Table 1), the risk-neutral probability that Alcoa will default in nine months is less than the risk-neutral probability that it will default in twelve. As such, the expected cash flows are no longer equal and therefore the two parties must have accounted for this discrepancy—otherwise Highbridge would not have agreed to the above CDS. In other words, given that the agreement was made, Citigroup must have paid an additional sum to Highbridge to compensate for the fact that she is paying the coupon for one-year coverage and is only receiving nine months.

In order to determine this sum, we need to calculate the fraction of the coupon she is unfairly paying, or the fraction of the coupon that accounts for the first three months of the year (January 1 to March 31). This fraction of the coupon payment is known as the **accrued coupon**.

$$\text{Accrued} = \frac{90}{360} \times \frac{1.0}{100} \times \$100,000 = \$250 \quad (21)$$

In order to determine the accrued coupon, we multiply the coupon (1%) by the notional amount (\$100,000) to get the coupon payment (\$1,000), and then we multiply the coupon payment by the fraction of the year that Highbridge did not receive coverage for—in this case, 90/360 or one fourth.

Note that we are dividing 90 by 360 instead of 365 in the above calculation. This particular CDS contract can be said to have a 30/360 **day count convention**, which means that the accrued coupon calculations are based on the assumptions that there are 30 days in a month and 360 days in a year. For example, if we are trying to find the number of days between date one,  $M_1/D_1/Y_1$ , and date two,  $M_2/D_2/Y_2$ , using the 30/360 convention, we use the following formula with exceptions listed below:

$$\text{Number of days} = 360 \times (Y_2 - Y_1) + 30 \times (M_2 - M_1) + (D_2 - D_1) \quad (22)$$

Exceptions:

1. If  $D_1$  is 31, assume that  $D_1$  is 30.
2. If  $D_2$  is 31 and  $D_1$  is 30 or 31, assume that  $D_2$  is 30.
3. If  $M_1$  is 2, and  $D_1$  is 28 (not in a leap year) or 29, assume that  $D_1$  is 30.

For example, according to this convention, if the simple one-period CDS agreement was initiated on February 28 (during a non-leap year), then we would calculate the number of days between 01/01/ $Y_a$  and 02/28/ $Y_a$  (where  $Y_a$  is a specific year) to be 59, since we consider February 28 to be the last day in a 30-day month (according to exception 3 above):

$$\text{Number of days} = 360 \times (0) + 30 \times (2 - 1) + (30 - 1) = 59 \quad (23)$$

If a CDS agreement was initiated on October 31, then we would calculate the number of days in between 01/01/ $Y_a$  and 10/31/ $Y_a$  (where  $Y_a$  is some year) to be 300. Note that it does not matter for this calculation if  $Y_a$  was a leap year or not.

$$\text{Number of days} = 360 \times (0) + 30 \times (10 - 1) + (31 - 1) = 300 \quad (24)$$

The purpose of this day count convention is to make calculations easier.

Since a \$250 cash flow from Citigroup to Highbridge compensates for the amount of the coupon that Highbridge is unfairly paying, we can model the expected cash flows for the agreement:

$$-\$1,000 + (P \times \$100,000) + \$250 = \$1,000 - (P \times \$100,000) - \$250 \quad (25)$$

Again, as neutral observer of the market (neither the buyer nor the seller), we are interested in how these expected cash flows affect  $P$ , the risk-neutral probability of default? Solving for  $P$ , we get:

$$P \times \$100,000 = \$750 \quad (26)$$

Therefore, the risk-neutral probability of default in this CDS is .75%. Note that this is different than that of the simple one-period CDS (1%). Note that in Table 1, we observed that the risk-neutral probability of default decreases as the contract matures.

In Section 2.2.1, we measured different aspects of a CDS over the life of the contract. Recall that we defined the mark-to-market value as the coupon that a prospective buyer would pay at a given time to own the protection for the remainder of the CDS. We could similarly measure the value of the accrued coupon over the life of the contract—or the accrued coupon value that Citigroup would have to pay a prospective buyer if she bought the protection from Highbridge at a given time in the contract. Consider the table below:

Date	Mark-to-market	$P\&L_H$	$P\&L_C$	Accrued Coupon
Jan 1	\$1,000	\$0	\$0	\$0
Mar 31	\$750	-\$250	\$250	\$250
Jun 30	\$500	-\$500	\$500	\$500
Dec 31	\$0	-\$1,000	\$1,000	\$1,000

Table 2: This table measures the mark-to-market value, the P&L for both parties, and the accrued coupon value over the life of the contract. Note that the mark-to-market values and accrued coupon values are the same for each date. In other words, the accrued coupon that accounts for the time passed is equal to the price that a prospective buyer would have to pay Highbridge for protection for the remainder of the CDS. The absolute value of each party's P&L is also equal to the accrued coupon value; the accrued coupon that accounts for the time passed is equal to the money the Citigroup has earned by providing protection for the time passed.

As we can see, the accrued coupon increases as the contract matures—or, in other words, the coupon that accounts for the time in the contract that has passed *increases* as the contract matures.

Note that accrued coupon only becomes relevant when the contract does not initiate on a coupon payment date. In the real-world CDS market, there are four dates each year—instead of, in our example, the one date of January 1—when coupon payments can be made. These dates are known as **roll dates** and comprise the following: March 20, June 20, September 20, and December 20. As we proceed with our simple CDS example, however, we'll continue with the assumption that January 1 is the one and only “roll date.”

There is also another related but different concept called accrued payment, which is paid by the protection buyer to the protection seller once a credit event happens. This payment is meant for the protection received between the last coupon date and the event date, since credit events might well not happen on a coupon date.

Both these “accrued” amounts serve as necessary adjustment to the payment of CDS transactions.

### *Non-standard Coupon and Big Bang Protocol*

In all of our CDS examples, we have assumed a coupon of 1%. We have done so because we

have been working with the same basic numerical figures: a risk-neutral probability of default of 1% and a notional amount of \$100,000:

$$-\$1,000 + ((P = 1\%) \times \$100,000) = \$1,000 - ((P = 1\%) \times \$100,000) \quad (27)$$

In other words, we have been considering cases in which Highbridge believes the probability of default to be at least 1%, and Citigroup believes the probability of default to be at most 1%. What if, however, Highbridge actually thought that the probability of default was 3%, and Citigroup thought it was 4%? Highbridge would want to make CDS agreements with risk-neutral probabilities of default of 3% or greater because if it's greater then she is paying the same coupon for a higher probability that she'll receive \$100,000. Citigroup would want to make CDS agreements with risk-neutral probabilities of default of 4% or smaller because if it's smaller then Citigroup is receiving the same coupon for a smaller probability that it'll have to pay \$100,000. The range of possible risk-neutral probabilities at which they would agree is 3% to 4%. Say they decide on 3%.

In such a case, the expected cash flows would only be equal if the coupon payment was \$3,000: 3% of the notional amount of \$100,000. As such, here is a case in which the coupon would not be 1%:

$$-\$3,000 + ((P = 3\%) \times \$100,000) = \$3,000 - ((P = 3\%) \times \$100,000) \quad (28)$$

In fact, that is a simplified example of how protection buyers and sellers determined coupon payments before April, 2009. Traders could theoretically enter CDS agreements with any coupon—say, .72% or 1.63%—because there were not any market conventions for coupon values. Each CDS had a specific **spread**—in this context, a specific coupon—that equated the two expected cash flows in the CDS. If a person was interested in buying CDS protection, she and the protection seller would refer to different CDS agreements by simply quoting the corresponding coupons.

One issue with this system was that traders could not easily trade CDS, since each CDS had a specific coupon. For example, investment banks like Citigroup often buy protection for one CDS and sell protection for another so as to offset each deal and **hedge** its risk. However, this became difficult to do when the CDS agreements had different coupons and thus different expected cash flows.

So, in 2009, a regulatory organization known as the International Swaps and Derivates Association (ISDA) introduced new CDS market conventions in North America which required



buyers and sellers to trade at coupons of 100 bps or 500 bps. This, along with a number of other regulatory changes, came to be known as the ‘Big Bang Protocol’. Since then, the spread of a CDS no longer signifies the coupon at which the CDS is traded; instead, the spread is the coupon that the CDS *would* be traded at in a market without standardized coupons—or the coupon that the CDS would have been traded at before 2009. In order to account for the discrepancy between the spread and the standardized coupon at which a CDS must trade, protection buyers and sellers use **upfront payments**, discussed in the next section.

### *Upfront Payment*

An **upfront payment**—a payment made from one party to the other at the beginning of a contract (in this case, January 1)—compensates for the difference between the spread that both parties *want* to trade at and the standardized coupon that they *have* to trade at. Suppose that, like in the previous example, both parties want to trade at a coupon of 3%. Clearly, they have to trade at a coupon of 1% or 5%—assume they choose 1%. They will only make a deal that acts *as if* they traded at a coupon of 3%.

Consider that both parties agree to the deal. How do we determine the value of the upfront payment?

First, review the expected cash flows we had in the previous section with a coupon of 3%:

$$-\$3,000 + ((P = 3\%) \times \$100,000) = \$3,000 - ((P = 3\%) \times \$100,000) \quad (29)$$

Since the difference between the protection buyer’s desired coupon payment (\$3,000 at 3%) and the payment she has to make due to CDS market convention (\$1,000 at 1%) is \$2,000, Highbridge’s upfront payment to Citigroup should be \$2,000. The best way of thinking about an upfront payment is to consider the difference in expected cash flows between the two parties. Since Highbridge’s expected cash flow is larger than that of Citigroup, the upfront payment ( $U$ ) is negative on the left side of the equation (Highbridge’s expected cash flow) and positive on the other side (Citigroup’s expected cash flow) so as to equate the two sides. (As before,  $C$  designates the coupon payment,  $P$  is the risk-neutral probability of default, and  $V$  is the notional value.)

$$\text{Premium leg} = \text{Protection leg} \quad (30)$$

$$-U - C + (P \times V) = U + C - (P \times V) \quad (31)$$

Plugging in known values:

$$-U - \$1,000 + (.03 \times \$100,000) = U + \$1,000 - (.03 \times \$100,000) \quad (32)$$

Simplifying:

$$2 \times \$3,000 = 2 \times \$1,000 + 2 \times U \quad (33)$$

$$\$3,000 - \$1,000 = U \quad (34)$$

This verifies that the correct upfront payment in this scenario is \$2,000.

That way, with an upfront payment, the two parties can trade with a coupon of 1% and a risk-neutral probability of default of 3%. On January 1, Highbridge would pay \$2,000 of the upfront payment to Citigroup and the \$1,000 coupon payment to Citigroup.

In Section 2.2.1, we measured different aspects of a CDS such as the risk-neutral  $P$  and the mark-to-market value over the life of the contract. Recall that we defined the mark-to-market value as the coupon that a prospective buyer would pay at a given time to own the protection for the remainder of the CDS. We could similarly measure the value of the upfront payment over the life of the contract—or the upfront payment that a prospective buyer would have to pay at a given time to own the protection for the remainder of the CDS. Consider the table below:

Date	$P$	Mark-to-market	Upfront Payment
Jan 1	3%	\$1,000	\$2,000
Mar 31	2.25%	\$750	\$1,500
Jun 30	1.5%	\$500	\$1,000
Dec 31	0%	\$0	\$0

Table 3: As the contract matures from January 1 to December 31, the value of the upfront payment (in a CDS where the coupon is 1%, the notional amount is \$100,000, and the risk-neutral probability of default is 3%) decreases from \$2,000 to \$0. Note that the true mark-to-market value of the CDS on, say June 30, would actually be the value in the mark-to-market column *plus* the value in the upfront payment column—because that is the total amount that the prospective protection buyer would have to pay to own the protection. However, for clarity, in this table we will view the values in the mark-to-market column as values that represent the coupon the prospective buyer would have to pay.

Earlier, we calculated the upfront payment using the following equation:

$$\text{Premium leg} = \text{Protection leg} \quad (35)$$

$$-U - C + (P \times V) = U + C - (P \times V) \quad (36)$$

We can verify that the upfront payment values in Table 3 are correct by inputting the risk-neutral probability of default as  $P$  and by inputting the mark-to-market value as  $C$ . For example, take June 30 (where, according to Table 3, the risk-neutral  $P$  is 1.5% and the mark-to-market value is \$500.)

$$-U - \$500 + (.0015 \times \$100,000) = U + \$500 - (.0015 \times \$100,000) \quad (37)$$

$$2 \times \$1,500 = 2 \times \$500 + 2 \times U \quad (38)$$

$$U = \$1000 \quad (39)$$

As such, on June 30, the value of the upfront payment is \$1,000—which is what Table 3 indicates, as well.

Note that in Equation (36),  $C$  and  $V$  are known or pre-determined values, since  $C$  is standardized by the ISDA and  $V$  is agreed upon by the protection buyer and seller. As such,  $P$  and  $U$  can be considered the unknown variables; one needs to know the value of  $P$  to determine the value of  $U$ , and vice versa.

Also, it is important to distinguish between two upfront payments: the **dirty upfront** and **clean upfront**. However, the the difference of the two upfront payments involves the concept of **accrued interest rates** which will be introduced in the N-period CDS example. For readers who are familiar with this concept, dirty upfront payment takes accrued interest rates into account, while clean upfront payment does not. Therefore, dirty upfront payment is the actual upfront payment that one party pays to the other.

### *One-Period Summary*

We have introduced a simple one-period CDS between a portfolio manager and Citigroup, and we have explored five complications that affect the expected cash flows: interest rate, recovery rate, accrued coupon, non-standard coupon and upfront payment.

In a real-world scenario, we'd have to look at *all* these factors at the same time, and come up with a way to model cash flows that take all factors into account.

Combining Equations (14), (18) and (36), we get a master equation that accounts for interest rate ( $i$ ), recovery rate ( $RR$ ), and upfront payment ( $U$ ). Accrued coupon can be calculated separately based on the date the CDS was initiated. (As before,  $C$  stands for the coupon payment,  $P$  for the probability of default, and  $V$  for the notional value.)

$$\text{Portfolio manager's expected cash flows} = \text{Citigroup's expected cash flows} \quad (40)$$

$$\text{Premium leg} = \text{Protection leg} \quad (41)$$

$$U - C + \frac{P \times V \times (1 - RR)}{(1 + i)} = -U + C - \frac{P \times V \times (1 - RR)}{(1 + i)} \quad (42)$$

Note that for any standard CDS contract, the two unknown variables are  $P$  and  $U$ , since  $C$  and  $RR$  are fixed by the International Swaps and Derivatives Association (ISDA)<sup>2</sup> (discussed in the next section), and  $V$  is agreed upon by the two parties (essentially fixed). As such, if we know  $P$ , we can calculate  $U$  and vice versa. At last, we will conclude this subsection with a numeric example. For instance, if we choose a standardized coupon of \$1000 and agree upon an effective interest rate of 0.1, a risk-neutral  $P$  of 1.5% and a recovery rate of 0.55, we can apply formula (42) to calculate the necessary upfront payment.

$$U - 1000 + \frac{1.5\% \times 100000 \times (1 - 0.55)}{(1 + 0.1)} = -U + 1000 - \frac{1.5\% \times 100000 \times (1 - 0.55)}{(1 + 0.1)}$$

Solving the equation, we get  $U = 386.37$ .

### 2.3. Two-Period Case

Thus far, we have described a simple one-period CDS with one coupon payment on January 1. However, as mentioned in Section 2.2.6, CDS have four coupon payments each year. Before we get into the exact mechanics of a four-period or more case, consider a simple two-period CDS that has a duration of two years and two coupon payments: one on January 1 of year one and one on January 1 of year two. The interest rate is 0%, and the probability of default is constant for both years of the contract. Like the simple one-period CDS, the notional amount is \$100,000 and the coupon is 1%. Note that the term "coupon" refers to the protection buyer's *annual* payment. So, the buyer will make two \$1,000 coupon payments, thus paying a total of \$2,000 over the two years. The protection buyer is still Highbridge and the protection seller is still Citigroup.

Consider that both parties agree to this deal.

The risk-neutral probability of default—the default probability that the deal implies both parties were willing to compromise on—can be calculated by equating the expected cash

<sup>2</sup>Note, ISDA does not set standard recovery rates for all reference entities.

flows. Consider the expected cash flows for year one of the CDS contract:

$$\text{Portfolio manager's expected cash flows} = -\$1,000 + (P \times \$100,000) \quad (43)$$

$$\text{Citigroup's expected cash flows} = \$1,000 - (P \times \$100,000) \quad (44)$$

The expected cash flows for Highbridge and for Citigroup are the same for year two.

In order to find the risk-neutral probability of default for each year, we must equate the expected cash flows:

$$-C + (P \times V) = C - (P \times V) \quad (45)$$

$$-\$1,000 + (P \times \$100,000) = \$1,000 - (P \times \$100,000) \quad (46)$$

$$P \times \$100,000 = \$1,000 \quad (47)$$

Thus, the risk-neutral probability of default for year one (or for year two, since the expected cash flows are the same) is 1%.

Now that we have considered the expected cash flows and risk-neutral probability of default for each year, we may want consider what those values for both years combined. What are the overall expected cash flows and the overall risk-neutral  $P$ ?

$$\text{Portfolio manager's expected cash flows} = -(\$1,000 + \$1,000) + (P \times \$100,000) \quad (48)$$

$$\text{Citigroup's expected cash flows} = (\$1,000 + \$1,000) - (P \times \$100,000) \quad (49)$$

In order to find the risk-neutral probability of default for both years combined, we need to equate the two expected cash flows:

$$-\$2,000 + (P \times \$100,000) = \$2,000 - (P \times \$100,000) \quad (50)$$

$$P \times \$100,000 = \$2,000 \quad (51)$$

The risk-neutral probability of default for the two-year contract is 2%. Note that the risk-neutral probability of default in the simple one-period CDS was 1%. The only reason that this probability is different is because Highbridge's coupon payment in the two-period case is double that of the one-period case.

Recall how we discussed in Section 2.2.1 that the probability of default within the context of a CDS decreases to 0% as the contract matures. Based on our calculations above, we can verify this relationship; after one year of the two-year contract, the probability of default has decreased from 2% at the beginning of the contract to 1% midway through the contract.

We will call the above CDS example the simple two-period CDS.

### *Simple Two-Period Case Complications*

Since this is a simplified example, we have excluded aspects of the CDS market that are worth noting.

1. We again assumed that both parties are risk-neutral, whereas this may not be the actual case. In reality, portfolio managers are prone to be more risk-averse and would be willing to pay a higher premium than entailed by the risk neutral assumption, while insurance providers can make a profit from this risk aversion.

2. In this example, we essentially assumed that we can just add probabilities: a probability of default of 1% during year one and a probability of default of 1% during year two equals a probability of default of 2% over the two years. But in real life, the probability of default over a big period may not just be a simple summation of different probabilities in separate small periods. For example, what is the probability of default of the two-year period if the probabilities of default in the first and second year are different?

3. In this example, we also assume that the interest rate is zero over the two years. But in real life, interest rates should always be taken into consideration. For example, the CDS pricing will change if there's a constant interest rate of 2% over the two years. Moreover, it might well be the case that the interest rate is not constant at all. In real life, the interest rates we use for CDS pricing often change over the period, so we should take that into consideration and apply different discount factors accordingly.

### *Non-constant Probability of Default*

In the simple two-period CDS, the overall probability of default is 2%. Since we conveniently assumed that the probability of default was constant over the two-year contract, each year had a probability of default of 1%. What if the overall probability of default was still 2%, but the probability of default is not constant—what if it's 2% the first year and 0% the second year?

In such a case, the expected cash flows for year one (where  $P = 2\%$ ) are as follows:

$$\text{Portfolio manager's expected cash flows} = -\$1,000 + ((P = 2\%) \times \$100,000) \quad (52)$$

$$\text{Citigroup's expected cash flows} = \$1,000 - ((P = 2\%) \times \$100,000) \quad (53)$$

Note that these expected cash flows are *not* equal. We will address this fact soon.

Now consider the expected cash flows for year two (where  $P = 0\%$ ):

$$\text{Portfolio manager's expected cash flows} = -\$1,000 + ((P = 0\%) \times \$100,000) \quad (54)$$

$$\text{Citigroup's expected cash flows} = \$1,000 - ((P = 0\%) \times \$100,000) \quad (55)$$

Clearly, the expected cash flows for Highbridge and Citigroup during year one are not equal, and the expected cash flows for Highbridge and Citigroup during year two are not equal, either.

However, note that in year one, Citigroup's expected cash flows are \$1,000 greater than that of Highbridge, and in year two, Citigroup's expected cash flows are \$1,000 less than that of Highbridge. As such, the overall expected cash flows (which are the same as Equation (50) because the overall risk-neutral probability of default is the same) are equal.

### *Non-constant interest rate*

First, let us give an example of a CDS which assumes a constant interest rate. Suppose the annual continuous risk-free rate is  $r = 0.1$  (such that the annual effective rate is  $e^{0.1}$ ). For simplicity purpose, further assume that the probable credit event can only take place at the end of the second year. Then redoing the two period case described by Equation (50), we end up having:

$$-\$1,000 - \frac{\$1,000}{e^{0.1}} + \frac{P \times 100,000}{e^{2 \times 0.1}} = \$1,000 + \frac{\$1,000}{e^{0.1}} - \frac{P \times 100,000}{e^{2 \times 0.1}}$$

Solving for  $P$ , we can get a risk-neutral  $P = 0.0232$ .

Further complication involves a non-constant interest rate. Now suppose the continuous risk-free rate takes a linear drop from 0.1 to 0.05 through the two years. In other words, let  $r = 0.1 - 0.025t$  where  $t \in [0, 2]$ . Then, Equation (50) needs to be rewritten as:

$$-\$1,000 - \frac{\$1,000}{e^{\int_0^1 0.1 - 0.025t dt}} + \frac{P \times 100,000}{e^{\int_0^2 0.1 - 0.025t dt}} = \$1,000 + \frac{\$1,000}{e^{\int_0^1 0.1 - 0.025t dt}} - \frac{P \times 100,000}{e^{\int_0^2 0.1 - 0.025t dt}}$$

Solving for  $P$ , we can get a new  $P = 0.0206$ , which is smaller than the risk-neutral probability in the constant interest rate example. Note again that as in previous cases, this risk-neutral probability is purely a mathematical concept which has nothing to do with the real probability of a credit event.

### Two-Period Case Over Time

Consider what happens to variables such as the cash flows of both parties, the risk-neutral  $P$ , the mark-to-market value, the spread, and the P&L of both parties as the two-year contract matures.

Date	$CF_H$	$CF_C$	$P$	MTM	Spread	$P\&L_H$	$P\&L_C$
Jan 1, $Y_a$	-\$1,000	\$1,000	2%	\$1,000	200 bps	\$0	\$0
Mar 31, $Y_a$	\$0	\$0	1.75%	\$750	175 bps	-\$250	\$250
Jun 30, $Y_a$	\$0	\$0	1.50%	\$500	150 bps	-\$500	\$500
Jan 1, $Y_b$	-\$1,000	\$1,000	1%	\$1,000	100 bps	-\$1,000	\$1,000
Mar 31, $Y_b$	\$0	\$0	.75%	\$750	75 bps	-\$1,250	\$1,250
Jun 30, $Y_b$	\$0	\$0	.50%	\$500	50 bps	-\$1,500	\$1,500
Dec 31, $Y_b$	\$0	\$0	0%	\$0	0 bps	-\$2,000	\$2,000

Table 4: This table displays how several variables—the cash flows (CF) of both parties, the risk-neutral  $P$ , the mark-to-market (MTM) value, the spread, and the profit and loss (P&L) of both parties—change as the two year contract matures.  $Y_a$  stands for year one and  $Y_b$  stands for year two. Note that this table shows these changes in the case that the reference entity—Alcoa—does not default. As such, the only cash flows are the two \$1,000 coupon payments on January 1 of each year. The risk-neutral probability of default decreases from 2% to 0% over the life of the contract, and the spread decreases from 200 bps to 0 bps. Note that the mark-to-market value, however, goes from \$1,000 to \$0 over the first year, and \$1,000 to \$0 over the second year.

Suppose that the spread (a.k.a. the coupon) is 160 bps (equivalent to 1.60%), and Highbridge and Citigroup have to trade at 100 bps because of CDS market conventions. On January 1, Highbridge pays an upfront payment to Citigroup that takes into account the extra 60 bps she doesn't have to pay as part of the standard coupon. Even though this payment happens once during the year, we can think about the value of this payment by considering what Citigroup earns every day. In return for providing protection each day, Citigroup earns a fraction of the total upfront payment. In other words, as the year progresses, the protection seller slowly makes money and the protection buyer slowly loses money. By the end of the year, both the upfront and the coupon payments have been made, effectively canceling each other out to produce one cash flow: a coupon payment of 1% from the protection buyer to seller. Thus, by the end of the year, the value of the upfront payment goes to zero since by the end of the year, since the cash flows cancel out as if there was never any upfront payment at all.

The way in which the value of the upfront payment drops over the life of the CDS contract



can be called the **pull to par**.

## 2.4. N-period Case

Having understood the basics of CDS, we can now consider the N-period case, which is more realistic than one- and two-period cases. Suppose the protection buyer now buys from Citigroup a CDS with duration of 5 years. Here, the duration of CDS is also called **tenor**, so we can say the contract has a tenor of five years in this case. And there are still four payment dates in one year, but different from the four dates assumed in Section 2.2. In reality, the four payment dates are Mar. 20, Jun. 20, Sept. 20, and Dec. 20. These dates can be also called **roll dates**. So assume the protection buyer buys the CDS on Jun. 20, 2014, then after five years, the CDS protection ends (or **matures**) on Jun. 20, 2019. The date on which a CDS matures is called **maturity date** or **end date**. Here, we pick the CDS trading date to be Jun. 20 to avoid complications such as upfront payments and **accrual amount**, which we will talk more about later.

Since there are four payment dates in one year and the tenor is five years, this is a 20-period example of CDS. Although the number of periods has increased dramatically from two to twenty, the central mathematics remains unchanged:

$$\text{Portfolio manager's expected cash flows} = \text{Citigroup's expected cash flows} \quad (56)$$

$$\text{Premium leg} = \text{Protection leg} \quad (57)$$

Assume a constant interest rate. A simple 20-period CDS will then have the below mathematic expression:

$$-\sum_{t=0}^{19} \frac{C}{e^{0.25tr}} + \frac{P \times V \times (1 - RR)}{e^{5r}} = \sum_{t=0}^{19} \frac{C}{e^{0.25tr}} - \frac{P \times V \times (1 - RR)}{e^{5r}} \quad (58)$$

where  $r$  stands for the constant continuous interest rate,  $V$  stands for notional value,  $RR$  stands for recovery rate, and  $C$  stands for coupon payment. Notice that we have assumed that the probable credit event can only happen at the end of five years, and hence the discount factor  $e^{-5r}$  for  $P \times V \times (1 - RR)$ . However, in reality, the probability of default is not constant over different periods. In fact, the default probability changes from time to time, so using a single factor to discount the protection leg is not accurate. For better modeling, we need to have a function, call it  $Q(t)$ , to represent the event probability at time  $t$ . Note that  $Q(t)$

is a cumulative distribution function whose density can be denoted as  $q(t)$ . With this, the buyer-seller equality can be improved as follows:

$$-\sum_{t=0}^{19} \frac{C}{e^{0.25tr}} + V \times (1 - RR) \times \int_0^5 e^{-rt} dQ(t) = \sum_{t=0}^{19} \frac{C}{e^{0.25tr}} - V \times (1 - RR) \times \int_0^5 e^{-rt} dQ(t) \quad (59)$$

Or:

$$-\sum_{t=0}^{19} \frac{C}{e^{0.25tr}} + V \times (1 - RR) \times \int_0^5 e^{-rt} q(t) dt = \sum_{t=0}^{19} \frac{C}{e^{0.25tr}} - V \times (1 - RR) \times \int_0^5 e^{-rt} q(t) dt \quad (60)$$

Further complications arise when we relax the assumption of a constant interest rate. So instead of using  $e^{-rt}$  to discount cash flow at time  $t$ , we will assume a more generic function,  $Df(t)$ , as our risk-free discount factor until time  $t$ . Moreover, since the protection buyer no longer has to pay coupons after a credit event, survival probability factors are needed to adjust the stream of coupon payments. Therefore, our final equality should be written as:

$$-\sum_{t=0}^{19} C \times Df(0.25t) \times S(0.25t) + V \times (1 - RR) \times \int_0^5 Df(t) dQ(t) \quad (61)$$

$$= \sum_{t=0}^{19} C \times Df(0.25t) \times S(0.25t) - V \times (1 - RR) \times \int_0^5 Df(t) dQ(t) \quad (62)$$

where  $S(t) = 1 - Q(t)$  is the survival probability until time  $t$ . This is our final version of buyer-seller equality for this section. Notice that other minor adjustment such as money market basis can still be made for finer modeling and will be introduced in the next section.

For conclusion, we point out that this buyer-seller equality has direct application in the pricing of CDS contract. The value of a CDS is just the difference in present value between the credit event leg and the premium leg, which can be expressed by the following formula:

$$PV(CDS) = PV(def) - PV(prem) = C \sum_{t_i} Df(t_i) S(t_i) - V \times (1 - RR) \int_0^T Df(t) dQ(t)$$

All these calculus complications in the N-period case are simply provided for reference. None are necessary nor for the understanding of latter sections.

### 3. Bloomberg and Markit

In this section, we will explore CDS pricing with the Bloomberg Terminal and the Markit CDS Calculator<sup>3</sup>, the two most frequently used tools for CDS calculations. Bloomberg L.P. is a global financial software and data company, and one of their key products is the Bloomberg Professional Service, also known as Bloomberg Terminal. The Bloomberg Terminal is a tool to monitor and analyze real-time market data for many financial products, and the service charges a substantial subscription fee. Markit is also a global financial company, and it specializes in credit derivative products.

### 3.1. Overview of CDS in Bloomberg Terminal



Figure 1: This is a Bloomberg Terminal screenshot that shows the details of an actual CDS transaction in Alcoa, priced on June 24, 2014 with 5 years of maturity. The screenshot consists of three sections: the Deal section (upperleft), Calculator section (bottomleft), and Market section (right side). The Deal section is where traders mostly input detailed information of the trade, such as **trade date** and **coupon**. The Calculator section shows the calculation results of the CDS contract after the trader has entered the trade information in the Deal section. Traders rely on these calculation results to make decisions on trades. The Market section shows information of the relevant market rates (such as LIBOR) that we concern as discount factors. This section also includes **recovery rate** and **default probability**.

<sup>3</sup>Available at <http://www.markit.com/markit.jsp?jsppage=pv.jsp>

### 3.2. Deal Section

Deal			
Buy	Notional	10 MM	USD
Contract		SNAC	
REF Entity	Alcoa Inc		
Debt Type	Senior	Restructuring	NR
REF Obligation	US013817AP64	RED Pair Code	014B98AD5
Trade Date	06/24/14	Trd Sprd (bp)	160.0000
1st Accr Start	06/20/14	Backstop Date	04/25/14
1st Coupon	09/22/14	Coupon (bp)	100.000
Pen Coupon	06/20/19	Day Cnt	ACT/360
		Freq	Q
Maturity	5Y	09/20/19	Pay AI True
			Date Gen I
Use Curve Recovery Rate	True		
Recovery Rate	0.40		
		Business Days	50
		Bus Day Adj	1
		Amrt	N

Figure 2: This is a figure of the Deal section cropped from Figure 1. Traders enter most of the CDS trade information in the Deal section, and Calculation section and Market section rely heavily on information provided in “Deal” section. Familiar terms are **notional amount**, **trade date**, **trade spread**, and **coupon**. Recall that trade spread is the coupon traders *would* have traded at before Big Bang Protocol standardized the coupon rate. **Maturity** means how long the protection contract lasts and when the coverage finishes. Here, the first slot “5Y” means the contract lasts 5 years, and second slot means that the contract will end on Sept. 20, 2019.

In the Deal section, **REF Entity** (Reference Entity) refers to the name of the company (Alcoa, in this case) and **RED Pair Code** is a Markit product that stands for Reference Entity Database. Each entity/seniority pair has a unique six-digit RED Pair Code that matches the first six digits of the nine-digit RED Pair Code, and a “preferred reference obligation,” which is the default reference obligation for CDS trades. A user can input either the six-digit RED Pair Code or the nine-digit RED Pair Code. The input “014B98” is the six-digit RED Pair Code for “Alcoa”. The label **Debt type**, marked as “Senior,” refers to the seniority of the debt. The **REF Obligation** (Reference Obligation) refers to the bond involved in the CDS, and it matches the ISIN. The label **Restructuring** refers to which term of debt restructuring we agree on: For example, U.S. traders usually do not accept debt restructuring as CDS payment, and debt restructuring is often considered as bankruptcy. Therefore, when trading CDS in U.S., we usually specify Restructuring to “NR,” which stands for “No Restructuring.” In Europe, however, traders usually consider certain types of debt restructuring acceptable and different from bankruptcy, so European traders usually specify Restructuring to “MMR”, which stands for Modified “Modified Restructuring”.

At the top left, we see the CDS contract type is **SNAC (Standard North American Contract)**, which is a convention that specifies how North American single-name CDS are supposed to trade. In European markets, CDS belong to the **STEC** category, or Standard European

Contract.

At the lower right, we see **Maturity** has two slots. The first slot is the **tenor**, or length, of the contract. This means that the buyer is protected for 5 years in this case. Tenor can also be denoted in months, e.g. "5M". The second slot is **Maturity Date**, or **end date**, of the contract. This refers to when the protection is over.

On the right side, **Trd Sprd (bp)** is just **spread**. Recall that spread is the "coupon" that we *trade* on before the Big Bang Protocol. Here the spread is equal to 160 basis points, but notice that spread can also be denoted in percentage. **Coupon (bp)** refers to **coupon** in basis points, which is equal to 100 basis points here. The protection buyer will have to make an upfront payment to account for the extra 60 basis points paid quarterly (15 bp per quarter) over a period of five years.

At the lower left, **Recovery Rate** is set to 0.40 and we use **Curve Recovery Rate** by default. On the bottom right, **trading conventions** are listed. For example, **Day Convention** ("Day Cnt") denotes what type of convention of dates we follow in this trading. Here, ACT/360 means that we assume there are 360 days in a year.

On the left, **Trade Date** is the date when we trade CDS; **First Accrual Start Date** ("1st Accr Start") is the previous **roll date** adjusted for weekdays before the Trade Date; **First Coupon Payment Date** ("1st Coupon") is the date when first coupon payment is paid. This is the first roll date adjusted for weekdays after our trade date. **Penultimate Coupon Payment Date** ("Pen Coupon") is the second last date when we pay our coupon payment. It is the weekday-adjusted roll date before the maturity date. Notice that the maturity date must **not** be adjusted, while other dates must be adjusted for businessdays. In other words, maturity dates can be on non-business days, but other dates cannot.

### 3.3. Calculator Section

Calculator		ISDA Standard Upfront Model (I)*	
Cash Settled On	06/27/14	Valuation Date	06/24/14
Cash Calculated On	06/27/14		
Price	97,125,417.55	Spread DV01	4,667.12
Principal	287,458	IR DV01	-75.65
Accrued (5 Days)	-1,389	Rec Risk (1%)	-325.01
Cash Amount	286,069	Def Exposure	5,712,542

Figure 3: This figure is the Calculation section cropped from Figure 2. We can see here that we are using ISDA Standard Upfront Model for the CDS pricing, and the valuation is done on the trade date while cash settlement date is three days after. Calculation section takes in the information entered by traders in Deal section and calculate some important results, such as **CDS price**, **principle amount**, and **cash amount**. Here, cash amount is the same as **upfront payment** ( $U$ ). Traders use these calculation results to make trading decisions and risk management.

At the top left, we choose **ISDA Standard Upfront Model** to calculate CDS pricing. **Principal**, or the mark-to-market value, of this contract is \$287,458. This amount is the sum of the expected present value of each coupon payment. If we account for the accrued amount which the protection buyer does not have to pay, the upfront value is \$286,069.

The ISDA protocol, since April 2009, specifies that all premium payments, by default, start on the roll date before the Trade Date. So if the Trade Date is June 24, 2014, the **Accrual Begin Date** is, by default, June 20, 2014. Now if the Accrual Begin Date is 4 days before the Trade Date, protection buyers would not want (and are not obligated) to pay interest for the 5 days they have not received protection for. **Accrued interest** can be calculated using the equation below:

$$\text{Accrued} = \frac{5}{360} \times \frac{1}{100} \times \$10,000,000 = \$1,389 \quad (63)$$

Now let's focus on the labels at the lower right: Spread DV01, IR DV01, Recovery Risk (1%), and Default Exposure. These four parameters indicate the change in upfront if small adjustments are made to the trade. For example, Spread DV01 measures how much upfront payment would change if spread was set a little higher. However, to understand these four concepts, we have to introduce **PV01** first.

#### *PV01*

**PV01** stands for "Present Value 01", which is the present value of a stream of 1 basis point payments. We can also use the formula below to calculate the principal amount (clean upfront payment) with PV01:

$$\text{Principal} = |\text{Coupon} - \text{Spread}| \times \text{PV01} \times \text{Notional} \quad (64)$$

PV01 can also be used to calculate the cash flows and risk measures of a CDS. It is sometimes referred to as the **CDS duration** or **risky duration**. Analytically, PV01 can be calculated by

$$\text{PV01} = \sum_t Df(t_i)S(t_i)B(t_i),$$

- $i$  = coupon index,
- $t_i$  = coupon date: date on which coupon payments are made
- $B(t_i)$  = day count fraction at  $t_i$ : fraction of the day on which the coupon payment is made upon, such as 360 or 365, depending on the day count convention of the CDS
- $Df(t_i)$  = discount factor until  $t_i$ ,
- $S(t_i)$  = survival probability until  $t_i$ ,

### Spread DV01

**Spread DV01** (also known as **Sprd DV01**, **Credit DV01**, **Spread Delta**, **DV01**) stands for "Spread Dollar Value 01." It measures the change of upfront payment if spread increases by 1 bp, and reflects the sensitivity of a CDS contract mark-to-market to a parallel shift in the term structure of the par spread. In other words, it reflects the risk duration of a CDS trade.

DV01 should always be positive for a protection buyer since she is short credit, and a rising spread is a sign of credit deterioration. Starting with PV01 and taking the derivative with respect to the spread gives us:

$$\begin{aligned} PV &= (S - C) \times \text{PV01} \\ \text{DV01} &= \frac{\partial PV}{\partial S} \\ &= \text{PV01} + (S - C) \frac{\partial \text{PV01}}{\partial S}, \end{aligned}$$

where  $S$  is the spread of the contract and  $C$  is the coupon. Both DV01 and PV01 are measured in dollars and are equal if the spread equals the coupon.

### IR DV01

**IR DV01** stands for “Interest Rate Dollar Value 01.” It measures the change in value of a CDS contract for a 1 bp parallel increase in the interest rate curve. IR DV01 is, typically, a much smaller dollar value than Spread DV01 because moves in overall interest rates have a much smaller effect on the value of a CDS contract than does a move in the CDS spread itself. Formula given below.

$$\text{IR DV01} = 0.0001 \frac{\partial PV}{\partial r} = 0.0001(S - C) \times \frac{\partial PV01}{\partial r}$$

A multiple of 0.0001 appears in the formula, because we are calculating the change of upfront payment with a 1 bp increase in the interest rate.

#### *Rec Risk (1%)*

**Rec Risk (1%)** stands for “Recovery Risk 1%.” It measures the change of upfront payment if the recovery risk increases by 1%. Formula given below.

$$\text{Rec Risk (1\%)} = 0.01 \frac{\partial PV}{\partial RR} = 0.01(S - C) \times \frac{\partial PV01}{\partial RR}$$

A multiple of 0.01 appears in the formula, because we are calculating the change of upfront payment with a 1% increase in the recovery rate.

#### *Default Exposure*

Default Exposure refers to the exposure to default of a CDS contract. It can be calculated with Recovery Rate, Notional Amount, and Principal as below:

$$\text{Default Exposure} = (1 - \text{Recovery Rate}) \times \text{Notional} - \text{Principal}.$$

### **3.4. Market Section**



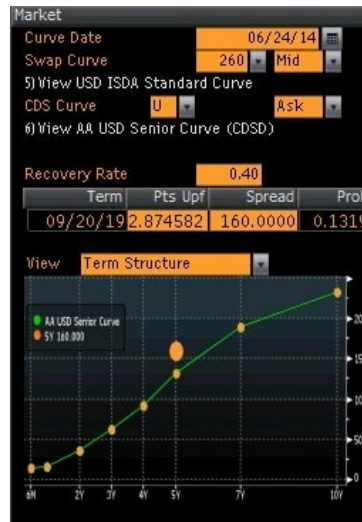


Figure 4: This figure is the Market section cropped from Figure 2. It shows information about market curve rates and CDS term structure. **Curve Date** is the date of LIBOR rates we use to price CDS. **Recovery rate** is the percent of a equity's value that can be recovered. **Points upfront** refers to the upfront payment as a percentage of the notional amount. **Spread** is the coupon rate we *would* have traded at before Big Bang Protocol standardized coupon rate. **Default probability** (or probability of default) is the risk-neutral probability of default that makes equal the potential cash flows from both buy and sell side of the CDS.

### *Curve Date*

**Curve date** is the date of interest rate we use to price CDS. As we recall, the trade date of this CDS trading is Jun. 24, 2014, and we see the Curve Date is also Jun. 24, 2014. However, this does not mean that we actually use the interest rates on Jun. 24, 2014; we are actually using interest rates on Jun. 23, 2014. For CDS pricing, we use the interest rates on the day before the trade date. And if the day before trade date is not a business day, we adjust to the nearest business day before. Therefore, if the trade date is Jun. 24, 2014, then we will use interest rates on Jun. 23, 2014 (Monday). If the trade date is Jun. 23, 2014, then we will use interest rates on Jun. 20, 2014 (Friday), since Jun. 22 is a Sunday. In Bloomberg Terminal, this adjustment is already taken into account, so the Curve Date on screen shows Jun. 24 while we are actually using interest rates on Jun. 23.

### *Default Probability*

As we did in the two-period case, we also have to account for the probability of default for each period. The probability of a company defaulting is different across time period, and we assume the probability is constant within a specific time period. The term structure of these probabilities are known as the **hazard rates** and the hazard rate function is piecewise constant. The probability of a company surviving till the end of period  $n$  is the probability that it

survives period  $n$ , conditional on it having survived the previous  $n-1$  periods. This is known as the **survival probability**. The survival probability function can be equated with the **hazard rates** function or the distribution of the company's risk-neutral probability of default as below,

$$S_t = 1 - Q_t = 1 - \int_0^T q_t dt$$

Here on the screenshot, the "prob" label refers to **default probability**. The non-default probability in a certain time-period, say from  $t_i$  to  $t_j$ , decays exponentially with multiplier determined by the current spread  $r$  at time  $t$ , time  $t$  itself, and the recovery rate. In other words, the default probability is non-constant but can be mathematically expressed as below:

$$\text{Non-Default Probability} \approx e^{\frac{rt}{1-RR}}, \quad (65)$$

where  $r$  is the spread,  $t$  is the time to maturity, and  $RR$  is the recovery rate. So the default probability at time  $t$  is:

$$\text{Default Probability} \approx 1 - e^{\frac{rt}{1-RR}}.$$

### *Term Structure*

A CDS term structure is similar to the interest rate curve in that it has the fair spreads of CDS contracts with different maturities. For liquid reference names, these rates are easily available. If the CDS is not liquid, we can use the underlying bond's price to determine the hazard rates. However, if there are only a couple of maturity dates available, we can assume a flat CDS curve. **Term structure** is shown in Market section (X-axis is maturity and Y-axis is spread). The green line with little yellow dots is the "AA USD Senior Curve", which means we are buying senior debt protection in USD of Alcoa Inc (stock symbol AA). The big yellow dot is the CDS contract we buy, here at 5-year maturity and 160 basis points of spread.

### 3.5. Overview of Markit CDS Calculator

**CDS Reference Entity And Contract Terms**

**Alcoa Inc.** (R) Required

**Trade Details**

Ticker/Company: AA (R) Red Code

Trade Date: 24-Jun-2014 (R) Buyer/Seller: Buyer (R)

Trade Level: 160.00 bp (R)

**CDS Contract Terms**

Maturity Date: 20-Sep-2019 (R) Running Cpn: 100 bps (R)

Notional (MM): 10 (R) Tier: SNRFOR (R)

Currency: USD (R) Recovery Rate: 40 % (R)

Counterparty: Restructuring: XR14

**Advanced Terms**

Cash Settlement: 3 Roll Code: F

Holiday Code: none Day Count Conv: A/360

Payment Freq: 3M Step In Date: 25-Jun-2014

Calculate Clear All

E-mail Results Send

Please note:  
All cash flows and default probabilities correspond to the trade level and trade maturity.  
Maturities correspond to: IMM dates

**Credit Curve**

Transform Method: Proportional

Maturity	Markit EOD	Transformed	Default Prob
6M		160.000 bps	1.9684 %
1Y		160.000 bps	3.2925 %
2Y		160.000 bps	5.8733 %
3Y		160.000 bps	8.3784 %
4Y		160.000 bps	10.8169 %
5Y		160.000 bps	13.1905 %
7Y		160.000 bps	17.7559 %
10Y		160.000 bps	24.1549 %
15Y		160.000 bps	33.7296 %
20Y		160.000 bps	42.0956 %
30Y		160.000 bps	55.7958 %

Recalculate

**Calculation Results**

Market Value	286.052
Cash Settlement	286.069
Accrued Days	5
Accrued Amt	1,388.89
Currency	USD

Details Show Cash Flows

	Transformed	User
Market Value	286.052	286.066
Clean Price	97.13 %	97.13 %
Cash Settlement	286.056	286.069
Accrued Days	5	5
Accrued Amt	1,388.89	1,388.89
Credit DV01	4.667	4.667
IR DV01	-76	-76

**Yield Curve**

Date: 23-Jun-2014

Snap: 1600 TimeZone: New York

Recalculate

Period	Type	Rate
1M	Deposit	0.1520 %
2M	Deposit	0.1963 %
3M	Deposit	0.2326 %
6M	Deposit	0.3253 %
1Y	Deposit	0.5471 %
2Y	Swap	0.6190 %
3Y	Swap	1.0630 %
4Y	Swap	1.4665 %
5Y	Swap	1.7930 %
6Y	Swap	2.0570 %
7Y	Swap	2.2725 %
8Y	Swap	2.4450 %
9Y	Swap	2.5880 %
10Y	Swap	2.7110 %
12Y	Swap	2.9085 %
15Y	Swap	3.1110 %
20Y	Swap	3.2915 %
25Y	Swap	3.3710 %
30Y	Swap	3.4100 %

**Interest Rate Conventions**

Spot Date: 26-Jun-2014 Swap DCC: 30/360

MM DCC: A/360 Swap Interval: 6M

Floating DCC: A/360 Holidays: none

Floating Interval: 3M Bad Day Conv: MF

Figure 5: This is a screenshot of the online Markit CDS Calculator. It shows the same trade as the above Bloomberg screenshot: a purchase of senior debt protection of Alcoa Inc with a tenor of five years on Jun. 24, 2014. The layout is divided into four sections: Contract section (left), Credit Curve section (center up), Calculation section (center down), and Yield Curve section (right). In Contract section, the fields are similar to those in Deal section in Bloomberg, except that we have to manually set holiday code (in Bloomberg this is automatically set according to users' input). Credit Curve section shows the term structure in a table, while in Bloomberg term structure is shown in a graph. Notice that we have the advantage of viewing default probabilities at different maturities in Markit, but advantage of viewing different spread levels in Bloomberg. our contract choice is highlighted in blue. The Calculation section is very similar to that in Bloomberg, with small differences explained below. The Yield Curve section shows the LIBOR rates (discount factor) we use, and notice that curve date is adjusted here.

#### Terminology Differences

On the upper left, **Trade Level** means the same as **spread**. We can present spread in either basis points ("bp") or percentages. In "CDS Contract Terms," **Running Cpn** just means **coupon**. In "Advanced Terms," **Holiday Code** refers to the holiday convention we are using. As we recall, CDS trading in JPY follows a different holiday convention than in other currencies. Therefore, we can select either "none" or "JPY" for "Holiday Code."

In "Calculation Results," **Cash Settlement** is the same as **Cash Amount** in Bloomberg, **Clean Price** is same as **Price**, the **Accured Amount** in Markit and Bloomberg has different signs (one is positive while the other negative), and **Credit DV01** is the same as **Spread DV01**.

### *Yield Curve Section and Discount Factor*

In the actual CDS world we would have to discount each of the 15 bp payments by the interest rates that apply for the same maturity. This is known as the discount factor. In the risk-neutral valuation framework, the risk free curve is the discount factor used in the calculation of the present value of CDS. In calculating the price of a CDS, we use the swap curve of the *previous day* (this will be further explained later in “Curve Date” section), and use the rates for the corresponding currency. So for CDS denominated in USD, we will use the US treasury rates, and for those in GBP we will use UK Government bond yields, and so on. If we look at Figure 5, we can see the yields for different treasury bonds at 4pm EST on June 23, 2014, which will be used for pricing the CDS of Alcoa.

The Yield Curve section in the above figure shows the risk-free rates with their day count conventions on June 23, 2014 at 4pm EST. These are the rates that will be used to price the CDS of Alcoa on June 24, 2014. **MM DCC**, **Floating DCC** and **Swap DCC** refer to the Day Count Conventions being used to calculate the different kinds of rates. A **Swap Interval** of ‘6M’ implies that interest is paid every six months or semi-annually. These conventions are different for each currency.

### *Risky Discount Factor*

Together, the discount factor and the survival probability are known as the **risky discount factor**. The discount factor is used in calculating the upfront value for a CDS maturing over  $n$  periods. Therefore the principal or mark-to-market value can be calculated using the formula below:

$$\sum_{i=1}^n |Coupon - Spread| \times (1 - q(t)) \times D(0, t(i)) \quad (66)$$

## 4. Using the **creditr** Package

In this section, we demonstrate the use of the **creditr** package and provide a series of examples. The **creditr** package implements the ISDA Standard Model, allowing users to value credit default swaps and calculate various risk measures associated with these instruments.

### 4.1. ISDA Standard Model

The ISDA has created the “Standard Model,” which allows market participants to calculate cash settlement from conventional spread quotations, convert between conventional spread

and upfront payments, and build the spread curve of a CDS.

The Standard Model allows market participants to convert between the par spread and the upfront payment, and compute the cash settlement amount for a standard contract.

The ISDA also standardizes the interest rates used by the Standard Model in valuing a CDS contract. There are two types of rates used in valuing a USD-denominated CDS contract: cash rates and swap rates. Cash rates are of the following maturities: one, two, three, and six month(s), and one year. These refer to the yields of zero-coupon bonds that have a maturity of less than one year; US treasuries with a maturity of one year or less are zero-coupon bonds. They are provided by the British Bankers' Association (BBA). Swap rates, or yields of coupon bond with a maturity of over one year, are of maturity 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 25, and 30 years, and are provided by ICAP (Markit 2013). The Standard Model follows the conventions below for interpolation of the entire USD yield curve:

- The day count convention (DCC) for money market instruments and the floating legs of the swaps is **ACT/360**.
- DCC for floating legs of the swaps is **30/360**.
- Payment frequency for fixed legs of the swaps is 6 months.
- Payment frequency for floating legs of the swaps is 3 months.<sup>4</sup>
- A business day calendar of weekdays (Monday to Friday) is assumed. Saturdays and Sundays will be the only non-business days.
- If a date falls on a non-business day, the convention used for adjusting coupon payment dates is **M** (Modified Following).
- Recovery Rate is the estimated percentage of par value that bondholders will receive after a credit event. It is commonly reported in percentage of notional value. CDS contracts for corporate bonds assume a 40% recovery rate for valuation purposes.

Currently, a market participant can conduct CDS-related calculations by using the **CDSW Calculator** on a Bloomberg Terminal or the Markit CDS Calculator<sup>5</sup>. The **creditr** package provides tools for valuing a single-name CDS contract. The default setting allows a user to value a USD-denominated CDS contract following the Standard Model as mentioned before. She can also specify her own set of parameters to customize the calculation.

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<sup>4</sup>See for details on floating and fixed legs calculation.

<sup>5</sup>The Markit CDS Calculator is available at

## 4.2. CDS Class

In the `creditr` package, we call the function `CDS` to construct an object of a class `CDS`.

```
> library(creditr)
> cds1 <- CDS(name      = "Alcoa",
+             RED       = "49EB20",
+             date      = as.Date("2014-06-24"),
+             tenor     = 5,
+             notional  = 10000000,
+             coupon    = 100,
+             spread    = 160)
```

Here the user enters the CDS contract with “Alcoa” as the underlying entity and sets the spread at 160 bps and the coupon at 100 bps. However, the valuation of a CDS contract requires neither the Reference Entity nor the RED Code. She does not have to know that information to use the `creditr` package. As shown below, as long as she inputs the same date, spread, and maturity information, the valuation of the contract will be the same.

```
> cds2 <- CDS(date      = as.Date("2014-06-24"),
+             maturity  = as.Date("2019-06-20"),
+             spread    = 160)
```

A user can call `summary` on a `cds1` to view essential information on the contract.

```
> summary(cds1)
```

Contract Type:	SNAC	Date:	2014-06-24
Entity Name:	Alcoa	RED:	49EB20
Currency:	USD	End Date:	2019-09-20
Spread:	160	Coupon:	100
Upfront:	286,069	Spread DV01:	4,667
IR DV01:	-75.64	Rec Risk (1 pct):	-330.19

In the summary output, it shows that the type of the CDS contract is “SNAC”. This means that the CDS contract follows the Standard North American Corporate (SNAC) CDS Contract specifications.<sup>6</sup> Date refers to the trade date which, in this example, is June 24, 2014. Entity

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<sup>6</sup>See for details.

Name refers to the entity name of the CDS contract which, in this example, is "Alcoa". The RED code is "48EB20" as specified by the user. currency ("USD" in this case) is the currency of the CDS contract with three options of "USD", "JPY" and "EUR". End Date is the same as maturity and is the date on which the CDS contract expires. Recall from previous sections that End Date can only fall on one of the four standard dates. In our example, the End Date is "2019-09-20". spread shows that the quoted spread for the contract is 160 bps and the coupon is 100 bps as shown in the Coupon field. Upfront indicates the dirty upfront payment in dollars or the cash settlement amount which, in this example, is \$286,069.

The remaining three items from the summary output are Spread DV01, IR DV01, and Rec Risk (1 pct). Spread DV01 is the increase in the upfront value when the spread increase by one basis point. In cds1, the Spread DV01 is \$4,667. IR DV01 is the increase in the upfront value when the interest rate on all maturities of the interest curve increase by a single basis point which, in our example, is \$-75.64. Rec Risk (1 pct) is the increase in the upfront value when the recovery rate increase by one percentage point. In cds1, the Rec Risk (1 pct) is \$-330.19.

Besides calling the summary method, one can type in the name of the CDS class object in the current R Session and obtain a full description of the CDS contract.

### Show Method

```
> cds1
```

#### CDS Contract

Contract Type:	SNAC	Currency:	USD
Entity Name:	Alcoa	RED:	49EB20
date:	2014-06-24		

#### Calculation

price:	97.13	Spread:	160
Principal:	287,458	Spread DV01:	4,667
Accrual:	-1,389	IR DV01:	-75.64
Upfront:	286,069	Rec Risk (1 pct):	-330.19
Default Prob:	0.1322		

### 4.3. CDS Pricing Related Functions

#### CS10

CS10 is a method which calculates the change in value of the CDS contract when the spread of the contract increases by 10%. CS10 takes in a CDS class object formed by calling the CDS function. The CS10 of `cds1` is \$25385.2. CS10 can also be calculated by inputting a data frame.

```
> x <- data.frame(date = as.Date(c("2014-04-22", "2014-04-22")),
+               currency = c("USD", "EUR"),
+               tenor = c(5, 5),
+               spread = c(120, 110),
+               coupon = c(100, 100),
+               recovery = c(0.4, 0.4),
+               notional = c(100000000, 100000000),
+               stringsAsFactors = FALSE)
> CS10(x)

[1] 56952.65 53430.42
```

#### *get\_rates*

`get_rates` obtains the LIBOR interest rate curve for a given currency on a given date, along with the day count convention for that currency. Recall that the actual curve date we are using is one business day before trade date.

```
> cds1.rates <- get_rates(date = as.Date("2014-06-24"), currency = "USD")
```

The output from the `get_rates` function is a data frame which contains rates of various expiries. They are directly fetched from a data frame included in the package, `rates`, or, if the requested date is not available, obtained from the Markit website based on the specifications ([Markit 2013](#)) (for foreign LIBOR rates before 2005, `get_rates` downloads the data from Federal Reserve website).

```
> cds1.rates

      date currency expiry   rate type
61418 2014-06-24   USD    1M 0.001520  M
```



61419	2014-06-24	USD	2M	0.001963	M
61420	2014-06-24	USD	3M	0.002326	M
61421	2014-06-24	USD	6M	0.003253	M
61422	2014-06-24	USD	1Y	0.005471	M
61423	2014-06-24	USD	2Y	0.006190	S
61424	2014-06-24	USD	3Y	0.010630	S
61425	2014-06-24	USD	4Y	0.014665	S
61426	2014-06-24	USD	5Y	0.017930	S
61427	2014-06-24	USD	6Y	0.020570	S
61428	2014-06-24	USD	7Y	0.022725	S
61429	2014-06-24	USD	8Y	0.024450	S
61430	2014-06-24	USD	9Y	0.025880	S
61431	2014-06-24	USD	10Y	0.027110	S
61432	2014-06-24	USD	12Y	0.029085	S
61433	2014-06-24	USD	15Y	0.031110	S
61434	2014-06-24	USD	20Y	0.032915	S
61435	2014-06-24	USD	25Y	0.033710	S
61436	2014-06-24	USD	30Y	0.034100	S

#### 4.4. Risk Related Functions

##### *rec\_risk\_01*

`rec_risk_01` is a function which takes a data frame of date, currency, tenor (or maturity), spread, coupon, recovery and notional and calculates the `rec.risk.01` values for each row.

Recall that **Rec Risk (1%)** stands for “Recovery Risk 1%.” It measures the change of upfront payment if the recovery risk increases by 1%. Formula of calculating Rec Risk (1%) is given below.

$$\text{Rec Risk (1\%)} = 0.01 \frac{\partial PV}{\partial RR} = 0.01(S - C) \times \frac{\partial PV01}{\partial RR}$$

```
> rec_risk_01(data.frame(date      = as.Date("2014-06-24"),
+                          currency = "USD",
+                          tenor     = 5,
```

```

+           spread = 160,
+           coupon  = 100,
+           recovery = 0.4,
+           notional = 10000000))

```

```
[1] -330.1858
```

### *IR\_DV01*

IR.DV01 is a function which takes a data frame of date, currency, tenor (or maturity), spread, coupon, recovery and notional and calculates the IR.DV01 values for each row.

Recall that **IR DV01** stands for “Interest Rate Dollar Value 01.” and it measures the change in value of a CDS contract for a 1 bp parallel increase in the interest rate curve. Formula of calculating IR.DV01 is given below.

$$\text{IR DV01} = 0.0001 \frac{\partial PV}{\partial r} = 0.0001(S - C) \times \frac{\partial PV01}{\partial r}$$

```

> IR_DV01(data.frame(date      = as.Date("2014-04-22"),
+                     currency = "USD",
+                     tenor    = 5,
+                     spread   = 160,
+                     coupon    = 100,
+                     recovery  = 0.4,
+                     notional  = 10000000))

```

```
[1] -73.72284
```

### *spread\_DV01*

spread\_DV01 is a function which takes a data frame of date, currency, tenor (or maturity), spread, coupon, recovery and notional and calculates the spread DV01 values for each row.

Recall that **Spread DV01** stands for “Spread Dollar Value 01” and it measures the risk duration of a CDS trade. Formula of calculating Spread DV01 is given below.

$$\begin{aligned}
 PV &= (S - C) \times PV01 \\
 DV01 &= \frac{\partial PV}{\partial S} \\
 &= PV01 + (S - C) \frac{\partial PV01}{\partial S},
 \end{aligned}$$

```

> spread_DV01(data.frame(date      = as.Date("2014-04-22"),
+                          currency = "USD",
+                          tenor    = 5,
+                          spread   = 160,
+                          coupon   = 100,
+                          recovery  = 0.4,
+                          notional  = 10000000))

[1] 4610.056

```

### *spread\_to\_pd*

`spread_to_pd` takes a data frame of spreads and calculates the probability of default using the ISDA Standard Model. Recall that assuming that non-default probability decays exponentially, the default probability is non-constant but can be mathematically expressed:

$$\text{Probability of Default} \approx 1 - e^{-\frac{r}{1-R}t},$$

and thus we can calculate probability of default using spread at time  $t$ .

```

> spread_to_pd(data.frame(date      = Sys.Date(),
+                          spread   = 160,
+                          tenor    = 5,
+                          recovery  = 0.4,
+                          currency  = "USD"))

[1] 0.1318644

```

## 5. Conclusion

In this paper, we have walked through three distinct but closely related parts regarding credit

default swaps: its theoretical background, general technology tools used for its pricing, and the credit R package.

For the first part, we started out with a simplistic account of property insurance, where we established an equality between protection buyer and protection seller and derived a risk neutral probability. From there, we began our study of one-period CDS by matching the cash flows from the protection leg with those from the premium leg, which is the same technique of “equating” as has been applied in the insurance case. Similarly, risk neutral probability was backed out and we have also observed the day-to-day changes in crucial measures as cash flows, risk neutral probability, mark-to-market, spread, P&L, etc. In the one-period case, mild complications regarding interest rate, recovery rate, default scenario, accrued coupon, day count convention and the Big Bang Protocol were introduced. With the one-period CDS as our foundation, we moved on to the two-period case where we have two designated coupon payments. In this subsection, we further complicated our model by dealing with non-constant probability of default, non-constant interest rate and looking at how a particular two-period example evolves over time. Finally, we came to the N-period case where multiple coupon payments are supported, and we even offered some formulas phrased in the calculus language for more conscientious readers. Complex as they may appear to be, those formulas still share the same spirit as our simple model in the initial subsection by assuming an equality of cash flows between counterparties and thus an ideal condition of non-arbitrage opportunities.

In Section 3: Bloomberg and Markit, we have reviewed two of most used software for pricing CDS contracts. We have noted that Bloomberg Terminal provides various financial services but charges a subscription fee, while Markit specializes pricing and regulating derivatives, and its Markit CDS Online Calculator is free. We then go into details of CDS pricing in Bloomberg Terminal. The CDS calculator interface is divided into three parts: the Deal section, the Calculator section, and the Market section. We have pointed out that the Deal section is where user specifies trade details, such as trade date, REF Entity, maturity, coupon and spread. These information provides inputs for Calculator sections for CDS pricing. Then we move to Calculator section, where calculation results are listed. Using the ISDA Standard Upfront Model, the Bloomberg CDS calculator returns the price, principal, and cash amount; these are important indicator of a CDS’s value and cash flows. The calculator also returns spread DV01 and IR DV01 and Rec Risk, which are indicators of a CDS’s risk and volatility. Along the introduction of these measurements, we have also provided the formulae for calculating them. Next we switch to Market section, which provides information on LIBOR rates and term structure. In addition, we have listed the formula for calculating default probability

decaying over time.

After introducing CDS window in Bloomberg Terminal, we start to present Markit CDS Calculator. Recall again that Markit Calculator is free while Bloomberg Terminal charges a monthly fee. The Calculator is divided into four parts, Contract section, Credit Curve section, Calculation section, and Yield Curve section. Contract section is basically the same as Deal section in Bloomberg Terminal; Credit Curve section is just the term structure in a table; Calculator section is highly similar to Bloomberg Calculator section; Yield Curve section is similar to Market section in Bloomberg. Then we move on to introduce some terminology differences, discount factors, and CDS indices.

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